Should you ask her out?

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1 Introduction

Let's say that you are feeling like going out tonight. You feel great and decide to go out for a drink with your friends. At the bar, you spot a (randomly selected) cute girl. You decide to start a conversation and since she seems to be really nice you want to ask her out on a date. Finally, you get the courage to do it. Unfortunately, she tells you she has a boyfriend. After this happened, your friends tell you that you should insist. After all, some girls just play hard to get. You struggle to decide what you should do about it. However, you're confident that your knowledge of probability should help you solve this dilemma.

2 Problem Formulation

Let's suppose that ω is the known proportion of girls that are single, λ is the known proportion of girls that would lie about their relationship status. Finally, for simplicity let's suppose that if the girl is single and you insist, then she will go out with you. In addition, you know that if the girl actually has a boyfriend, then even if you insist she will not go out with you.

Define $\boldsymbol{B}$ as the event 'the girl has a boyfriend', $\boldsymbol{A}$ as the event 'she answers that she has a boyfriend', and $\boldsymbol{D}$ as the event that she will go out with you. Furthermore, define $\boldsymbol{I}$ as the event 'you insist'. The probabilistic statements that describe your situation are:

$$\Pr(\boldsymbol{B}) = 1 - \omega \tag{1}$$

$$\Pr(\boldsymbol{A} \mid \boldsymbol{B}^c) = \lambda \tag{2}$$

$$\Pr(\boldsymbol{A} \mid \boldsymbol{B}) = 1 \tag{3}$$

$$\Pr(\boldsymbol{D} \mid \boldsymbol{I}, \boldsymbol{B}^c, \boldsymbol{A}) = \Pr(\boldsymbol{D} \mid \boldsymbol{I}, \boldsymbol{B}^c) = 1 \tag{4}$$

$$\Pr(\boldsymbol{D} \mid \boldsymbol{I}^c, \boldsymbol{A}) = 0 \tag{5}$$

$$\Pr(\boldsymbol{D} \mid \boldsymbol{B}) = 0 \tag{6}$$

3 Optimization Problem

You face the decision of whether insisting or not. That is, to choose $i \in \{\boldsymbol{I}, \boldsymbol{I}^c\}$ given that you observed $\boldsymbol{A}$, so you maximize the expected utility of your decision:

$$\max_{i \in \{\boldsymbol{I}, \boldsymbol{I}^c\}} \quad \mathbb{E}\left[\mathbb{U} \mid i, \boldsymbol{A}\right]$$

where:

$$\mathbb{U}\left(d, i\right) \quad = \quad u \cdot \mathbb{1}_{\boldsymbol{D}}(d) - c \cdot \mathbb{1}_{\boldsymbol{I}}(i)$$

and:

$$u > c > 0$$

3.1 Don't Insist

First, let's notice that following equation 6, if you don't insist there is no chance that she will go out with you. Therefore,

$$\mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}^c, \boldsymbol{A}\right] \quad = \quad 0$$

3.2 Insist

Using the fact (Bayes Theorem) that:

$$\Pr(\boldsymbol{B}^c \mid \boldsymbol{A}) \quad = \quad \frac{\Pr(\boldsymbol{A} \mid \boldsymbol{B}^c)\Pr(\boldsymbol{B}^c)}{\Pr(\boldsymbol{A} \mid \boldsymbol{B}^c)\Pr(\boldsymbol{B}^c) + \Pr(\boldsymbol{A} \mid \boldsymbol{B})\Pr(\boldsymbol{B})}$$

and the assumptions and results stated before, you conclude that:

$$\mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}, \boldsymbol{A}\right] \quad = \quad \frac{u \cdot \lambda \cdot \omega}{1 - (1 - \lambda) \cdot \omega} - c$$

3.3 Results

Following the expected utility theory, you realize that the decision is quite easy. You should insist when:

$$\begin{aligned} \mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}, \boldsymbol{A}\right] \quad &> \quad \mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}^c, \boldsymbol{A}\right] \\ \frac{u \cdot \lambda \cdot \omega}{1 - (1 - \lambda) \cdot \omega} - c \quad &> \quad 0 \end{aligned}$$

3.3.1 Sensitivity Analysis

A remarkable result is that if λ was equal to 0, then it would never make sense to insist. An explanation of this fact is that if women never lied, then the signal $\boldsymbol{A}$ would provide perfect information about their relationship status. In addition,

$$\frac{\partial \mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}, \boldsymbol{A}\right]}{\partial \lambda} \quad = \quad \frac{u \cdot \omega \cdot (1 - \omega)}{(1 - (1 - \lambda) \cdot \omega)^2} \quad > \quad 0$$

An interesting consequence is that the more likely it is that women lie, the better it is for you to insist. Whoa! the lying strategy backfired for them.

4 How much do women lie?

In a world of perfect information, you would be done by now (actually, you would have been done since the beginning of the paper). Nonetheless, you're aware that much of the argument used before relies on the fact that λ is known. This, however, doesn't seem to be quite realistic. After all, you only have a vague idea of how much women lie.

4.1 Bayesian Inference

From now on, you decide to adopt the Bayesian interpretation of probability according to which probabilities encode degrees of belief about events in the world and data are used to strengthen or weaken those degrees of belief.

4.1.1 Prior Knowledge

Now, let's suppose that λ is no longer known. Instead of that, you have the (prior) belief that:

$$\lambda \quad \sim \quad Beta(\alpha, \beta)$$

Therefore, the probability density function of λ is:

$$f(\lambda) \quad = \quad \frac{\lambda^{\alpha-1} \cdot (1-\lambda)^{\beta-1}}{\mathbb{B}(\alpha, \beta)}$$

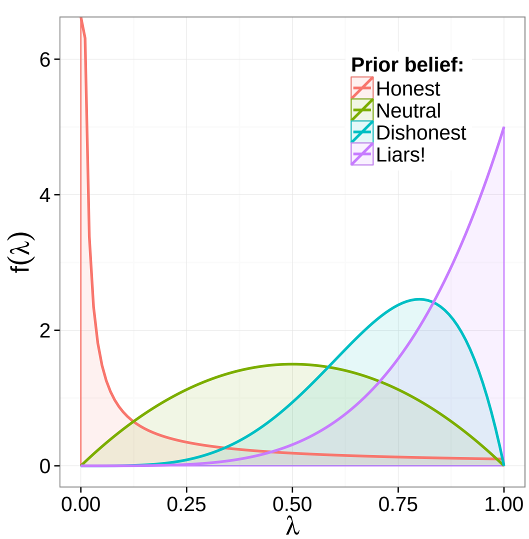


Figure 1: Different alternatives for the prior distribution.

4.1.2 Likelihood Function

In addition, you have observed what happened when you insisted to a sample of similar girls (drawn from the same distribution). In your sample, you have that from n girls to whom you have insisted, x agreed to go out on a date with you. You also recognize that for a given λ, the likelihood of this happening is:

$$f(x \mid \lambda) \quad = \quad \binom{n}{x} \cdot \lambda^x \cdot (1-\lambda)^{n-x}$$

4.1.3 Inference

Now, we use again Bayes Theorem, in it's Bayesian interpretation, where:

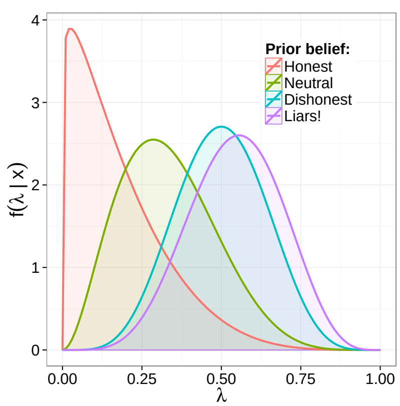
$$\text{Posterior} \quad \propto \quad \text{Likelihood} \cdot \text{Prior}$$

to obtain:

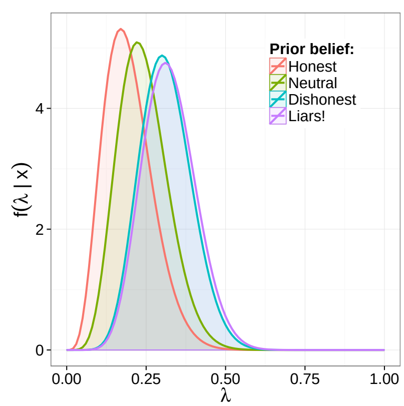
$$f(\lambda \mid x) \quad \propto \quad \lambda^{\alpha+x-1} \cdot (1-\lambda)^{\beta+n-x-1}$$

Therefore, you conclude that with the available information, the posterior distribution of λ is:

$$\lambda \quad \sim \quad Beta(\alpha + x, \beta + n - x)$$



(a) Posterior when $n = 5$ and $x = 1$.



(b) Posterior when when $n = 25$ and $x = 5$.

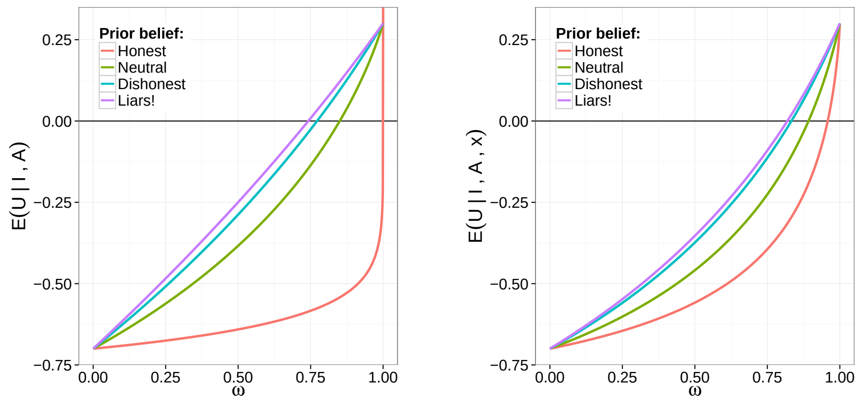
Figure 2: Different alternatives for the posterior distribution.

4.2 New solution

Now, the expected value of insisting has changed a little. The new expected value:

$$\mathbb{E}\left[\mathbb{U} \mid \boldsymbol{I}, \boldsymbol{A}, x\right] \quad = \quad -c + u \cdot \int_0^1 \frac{\lambda \cdot \omega}{1 - (1-\lambda) \cdot \omega} \cdot f(\lambda \mid x) \quad d\lambda$$

which can be evaluated numerically. Actually, *Mathematica* provides a closed solution in terms of the regularized hypergeometric function, which can be evaluated numerically.[1] Alternatively, it can be approximated simulating realizations of a Beta distribution.



(a) A priori.

(b) After observing $n = 5$ and $x = 1$.

Figure 3: Expected value of insisting when $u = 1$ and $c = .7$

5 Final Remarks

There you go. Now you have all the information and analysis that you need to take the best decision. Nonetheless, if you still find the model quite simplistic for your situation, there are some alternatives you can try. A natural extension of this simple model is to change equation 4 for the following:

$$\Pr(\boldsymbol{D} \mid \boldsymbol{I}, \boldsymbol{B}^c, \boldsymbol{A}) = \Pr(\boldsymbol{D} \mid \boldsymbol{I}, \boldsymbol{B}^c) = \theta \tag{7}$$

where θ would reflect the willingness of girls without boyfriend to go out with you after you insist. Again, you would have to perform inference over θ. Furthermore, the inference would have to be done simultaneously.

Finally, don't forget that the easiest way to actually find someone to go on a date with you is to stop reading this and go out.

[1] http://reference.wolfram.com/mathematica/ref/Hypergeometric2F1Regularized.html